Logarithms

Other Handouts:

- Scientific notation • Units
- Significant Figures • Graphing
- Ratio and Proportion
- Review of Number

The order of magnitude of a power of 10 is called the **logarithm** of that power of 10.

So for 10^6, its logarithm is **6**. We would write this as $\log_{10}\ 10^6 = 6$ (the **base** of this logarithm is the base of the power, i.e. 10) base

We can consider powers of other numbers also. For instance $2^3 = 2x2x2 = 8$ and we would say that the logarithm to the base 2 of 8 is **3** (the power of 2 that gives 8), or $\log_2\ 8 = 3$.

Examples

$\log_{10}\ 100 = 2$ because $10^2 = 100$

$\log_{10}\ 0.0001 = -4$ because $10^{-4} = 0.0001$

$\log_{10}\ {}^1/_{1000} = -3$ because $10^{-3} = {}^1/_{1000}$

$\log_{10}\ 1 = 0$ because $10^0 = 1$

$\log_4\ 16 = 2$ because $4^2 = 16$

$\log_2\ 16 = 4$ because $2^4 = 16$

$\log_3\ 9 = 2$ because $3^2 = 9$

$\log_5\ {}^1/_5 = -1$ because $5^{-1} = {}^1/_5$

Practice Problems – Logarithms

1. Find the following logarithms

(a) $\log_{10}\ 10{,}000$ (b) $\log_{10}\ 0.001$ (c) $\log_2\ 8$ (d) $\log_2\ {}^1/_{32}$

(e) $\log_5\ 125$ (f) $\log_9\ {}^1/_{81}$ (g) $\log_{64}\ 8$ (h) $\log_{10}\ 10$

2. Find the value of x in the following.

(a) $\log_{10}\ x = 5$ (b) $\log_{10}\ x = -2$ (c) $\log_4\ 16 = x$ (d) $\log_{25}\ x = {}^1/_2$

(e) $\log_x\ 27 = 3$ (f) $\log_9\ {}^1/_9 = x$ (g) $\log_5\ 625 = x$ (h) $\log_{10}\ 0.000\ 001 = x$

Logarithms on your calculator

Somewhere on your calculator there will be a button marked 'log'. This is the *common* logarithm to the base 10. Depending on the design of your calculator, you might have to press log first and then enter the number; or you might have to press shift 10^x to access the log function.

3. On your calculator find the logarithm to the base 10 of 314. Now find the logarithm of 3.14. What do you notice? What is the order of magnitude of 314?

Using the order of magnitude to help you, and without using your calculator, find

(a) $\log_{10}$ 314,000 (b) $\log_{10}$ 3.14×10^{27} (c) $\log_{10}$ 0.00314

4. The acidity of a solution is measured on the pH scale. The pH is the negative of the logarithm to the base 10 of the hydrogen ion concentration $[H^+]$ of the solution. That is,

$$pH = -\log_{10} [H^+]$$

(a) Pure water has a hydrogen ion concentration of $[H^+] = 10^{-7}$ moles/litre. Find its pH.

(b) If the pH of a solution is 2 what is its hydrogen ion concentration $[H^+]$?

Solutions to Practice Problems - Logarithms

1. 4 , -3 , 3 , -5 , 3 , -2 , ½ , 1 2. 100,000 , 0.01 , 2 , 5 , 3 , -1 , 4 , -6

3. $\log_{10}$ 314 = 2.497 $\log_{10}$ 3.14 = 0.497 order of magnitude of 314 is 2

(a) 5 + 0.497 = 5.497 (b) 27 + 0.497 = 27.497 (c) -3 + 0.497 = -2.503

4.(a) 7 (b) 0.01 moles /litre