

Working with Formulas

Working with formulas

In chemistry, like most sciences, you will work a lot with formulas full of symbols. At first this can be quite intimidating, but once you have figured out how to work with formulas you'll find they are a great help to you in solving real-life problems in medicine, conservation, agriculture and industry. There are chemical formulas like CO_2 for carbon dioxide and mathematical formulas like $PV = nRT$ that are used to find the values of variables. There are chemical equations like $\text{HCl} + \text{H}_2\text{O} \rightarrow \text{Cl}^- + \text{H}_3\text{O}^+$, and mathematical equations like $2x + 3 = 12$. Formulas often have more than one variable; simple equations have only one. However the strategies to manipulate both formulas and equations are the same and will be the focus of this topic.

Substitution First you need to get the hang of substituting numbers into formulas.

Example 1 To find the volume of a box we can use the formula $V = \ell \times b \times h$ (often written as ℓbh) where ℓ = length, w = width, h = height.

Write the numbers you know in place of the variables in the formula.

Suppose $\ell = 10\text{m}$, $b = 4\text{m}$, $h = 2\text{m}$, then

$$V = \ell \times b \times h$$

i.e.

$$V = 10 \times 4 \times 2$$

i.e.

$$V = 80\text{m}^3$$

So the volume of the box is 80m^3 .

Example 2 A taxi fare may be worked out using the formula $c = f + rd$

where f is the flagfall, r is the cost per kilometre and d is the number of kilometres travelled.

Find the taxi fare if $f = \$3.00$, $r = \$1.20$, $d = 6\text{km}$.

So

$$c = 3.00 + 1.20 \times 6$$

i.e.

$$c = 10.20$$

Therefore the cost of the taxi is $\$10.20$.

Practice Exercises : Working with formulas

Exercise 1 Substitute the values of $b = 1.3$, and $c = 6.9$ into these formulas to find the value of a .

Note '/' means divide.

(i) $a = b + c$	(iv) $a = c - b$
(ii) $a = b/c$	(v) $a = 3c - 2b$
(iii) $a = c/b$	(vi) $a = b^2 + c$

Substitution and Solving formulas You can calculate any of the variables in a formula as long as you know what all the others are.

Example 3 $V = a b c$

Suppose you know the values of $V = 36$, $a = 3$ and $b = 2$ - but don't know c .

We remember $V = a \times b \times c$ and substitute the values we do know into the formula. In other words...

$$30 = 3 \times 2 \times c \quad \text{i.e.} \quad 30 = 6 \times c$$

So it looks like c must equal five, because $30 = 6 \times 5$.

Manipulating Formulas You can perform any operation on a formula, or equation if you 'do the same to both sides'.

Example 4 Using the taxi formula again $c = f + rd$, where $c = 15.00$, $f = 3.00$, $r = 1.20$, substitute the numbers you know and solve for d .

$$c = f + rd$$

i.e.

$$15 = 3.00 + 1.20 \times d$$

Take 3.00 from each side : $12.00 = 1.20 \times d$

Divide each side by 1.20 : $10 = d$

so the ride was 10 kilometres.

Example 5 Use the formula $5a = b/c$ to find b, given that a = 6 and c = 3:
i.e. $a \times 5 = \frac{b}{c}$ i.e. $6 \times 5 = \frac{b}{3}$

We want to get rid of the 3 on the right hand side, so as to isolate b as the subject of the equation. Since the right side is divided by 3, we will do the opposite and multiply. But to follow our rule, we must also then multiply by 3 on the left hand side as well:

$$3 \times 6 \times 5 = \frac{3 \times b}{3}$$

Now the two 3's on the right side cancel:

$$3 \times 6 \times 5 = b \quad \text{i.e.} \quad 90 = b$$

Practice Exercises continued

Exercise 2

Substitute the values a = 6 and c = 2 into these formulas, and determine the value of b.

(i) $a = b + c$	(iii) $a = 3c + b$	(v) $a = 8c - 2b$
(ii) $a = b/c$	(iv) $a = b - c$	(vi) $a = 2b + c$

Exercise 3

1. The pressure of oxygen in kPa in a resuscitation cylinder is related to the temperature in degrees Celsius:

$$\text{pressure} = 5.5 \times \text{temperature} + 1500$$

i.e. $P = 5.5T + 1500$

Work out what the temperature would be when the pressure is 2.0×10^3 kPa.

2. This formula is used to calculate how concentrated or dilute a solution is:

$$\text{concentration} = \frac{\text{mass of dissolved substance}}{\text{volume of solvent}} \quad \text{i.e.} \quad C = \frac{M}{V}$$

a) Work out how much salt you would need to make up 1.2 L of electrolyte drink with a salt concentration of 0.38 gL^{-1} . Give the answer in scientific notation to an appropriate number of significant figures.

b) Work out what volume of water you would need to make up an electrolyte drink to a magnesium concentration of 496 mg.L^{-1} , using 56 g of magnesium ions. Give the answer in scientific notation to an appropriate number of significant figures.

3. This equation is used to work out the space taken up by a gas under pressure:

$$\text{pressure} \times \text{volume of gas} = \text{no. moles of gas} \times R \times \text{temp} \quad \text{i.e.} \quad PV = nRT$$

R is the universal gas constant, a special number that relates the volume (V) and number of moles of a gas (n) to its temperature (T) and pressure (P).

a) Using the figures below, work out the pressure of the gas. Give the answer to two significant figures.

Volume of gas = 10 L

Amount of gas = 2.0 moles

Temperature = 260 Kelvin

$R = 0.082 \text{ L.atmospheres/Kelvin.mole}$

b) Using this next set of numbers, work out the amount of gas in moles. Give the answer in scientific notation to an appropriate number of significant figures.

Pressure = 1.00 atmospheres

Volume = 5.00 L

Temperature = 298 Kelvin

$R = 0.0821 \text{ L.atmospheres/Kelvin.mole}$

More tips on rearranging formulas

Actually, you don't even need to know any of the numbers to rearrange an equation. Take this one:

$$\text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad D = M / V$$

If you want to change the equation so that mass is the subject, multiply both sides by volume.

$$\text{density} \times \text{volume} = \frac{\text{mass} \times \text{volume}}{\text{volume}}$$

We don't even know what the volume is – but if any number is divided by itself, it must cancel:

$$\text{density} \times \text{volume} = \frac{\text{mass} \times \cancel{\text{volume}}}{\cancel{\text{volume}}}$$

So the formula is,

$$\text{density} \times \text{volume} = \text{mass} \quad \text{or} \quad D V = M$$

Practice Exercises continued

Exercise 4

Rearrange these formulas to make 'c' the subject of each one:

(i) $a = b + c$	(iii) $a = 3c + b$	(v) $a = 8c - 2b$
(ii) $a = 2b + c$	(iv) $a = b - c$	(vi) $a = b/c$

Exercise 5

1. Examine each of the formulas in Exercise 4. Assuming 'c' was fixed at 10, how would 'a' change if 'b' were increased? Would 'a' increase or decrease?

2. Rearrange this formula $PV = nRT$

a) to make 'P' the subject

b) to make 'T' the subject

c) to make 'n' the subject

3. Rearrange this formula

$$y = \frac{a-b}{c}$$

a) to solve for 'a'

b) to solve for 'b'

c) to solve for 'c'

Solutions to Practice Exercises: Working with Formulas

Exercise 1

(i) 8.2

(ii) 0.19

(iii) 5.3

(iv) 5.6

(v) 18

(vi) 8.6

Exercise 2

(i) 4

(ii) 12

(iii) 0

(iv) 8

(v) 5

(vi) 2

Exercise 3

1. 91 degrees Celsius (to 2 s.f.)

2.(a) 4.6×10^{-1} g (2 sig. figures)

(b) 1.1×10^2 L

3.(a) 4.3 atmospheres (2 sig. figures)

(b) 2.04×10^{-1} moles (3 sig. figures)

Exercise 4

(i) $c = a - b$

(ii) $c = a - 2b$

(iii) $c = (a - b)/3$

(iv) $c = b - a$

(v) $c = (a+2b)/8$

(vi) $c = b/a$

Exercise 5

1. (i) increase (ii) increase (iii) increase (iv) increase (v) decrease (vi) increase

2. a) Divide by V on both sides; cancel; final

equation is:
$$P = \frac{n \times R \times T}{V}$$

b) Starting with this equation,

$$P \times V = n \times R \times T$$

Divide both sides by n; cancel; divide both sides

by R; cancel; final equation is:
$$\frac{P \times V}{n \times R} = T$$

c) Starting with this equation,

$$P \times V = n \times R \times T$$

Divide both sides by R; cancel; divide both sides

by T; cancel; final equation is:

$$\frac{P \times V}{R \times T} = n$$

3. a) Multiply by c on both sides; cancel; add b on both sides; cancel; final equation is:

$$a = y \times c + b$$

b) Starting with this equation,

$$y = \frac{a - b}{c}$$

Multiply both sides by c; cancel; subtract a from both sides; cancel; final equation is:

$$b = y \times c - a$$

c)

$$c = \frac{a - b}{y}$$