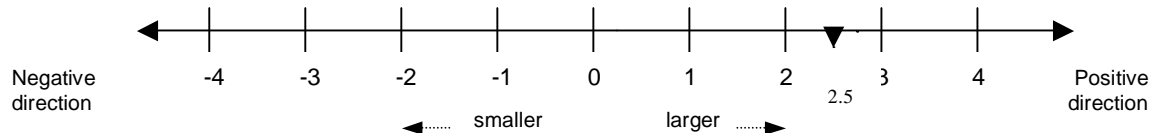


Review of Number

Other Handouts:

- Scientific notation • Units
- Significant Figures
- Ratio and Proportion
- Logarithms • Graphing

Real numbers can be represented as points on a real number line.



{ 1, 2, 3, 4, 5, 6, 7, ... } is the set of **natural** numbers or **counting** numbers.

{ 0, 1, 2, 3, 4, 5, 6, 7, ... } is the set of **whole** numbers. Allowing for positive and negative directions,

{ ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ... } is the set of **integers**.

Rational numbers can be written as the quotient of two integers, with a non-zero denominator, e.g. $\frac{2}{3}$, $-1\frac{1}{4}$, 15.3, and so can be expressed as a fraction or as a terminating or repeating decimal.

Irrational numbers are all the numbers on the real number line that are not rational, e.g. $\sqrt{2}$, $-\sqrt{10}$, π , $\sqrt[5]{2}$

Every real number is a distance from the point 0 on the number line and its sign (negative or positive) indicates the direction along the line. So -5 is 5 units from 0 in the negative direction, which is in the opposite direction to $+5$. A negative sign is used to indicate a change to the **opposite** direction, so $-(-5)$ is $+5$.

The **absolute value** $|x|$ of a number x is its distance from 0 along the real number line (ignoring its sign). The absolute value is always positive. So $|-4| = 4$

A **prime** number is a natural number, greater than 1, that cannot be divided exactly by any natural numbers other than 1 and itself, e.g. 2, 3, 5, 7, 11, 13. **Composite** numbers are natural numbers that are not prime, and so have other divisors, e.g. 12 is divisible by 1, 2, 3, 4, 6 and 12.

Practice Problems – Real numbers

1. List all the numbers in the set { -9.001 , $-\sqrt{7}$, 10, $-1\frac{1}{4}$, 0, $\sqrt{5}$, 3, 8.94, $-\frac{3}{5}$, 7, -2 , 21, 37 } that are

(a) natural	(b) whole	(c) integers	(d) negative real numbers
(e) rational	(f) irrational	(g) prime	(h) composite

2. Use a number line to decide whether the following are true or false.

- | | | | |
|--------------------------------|----------------------------|-------------------------------|-----------------------------|
| (a) $-1\frac{1}{4} < -1$ | (b) $1 > -\frac{1}{2}$ | (c) $ 2 = 2$ | (d) $7 - -1 = 6$ |
| (e) $(-2)^3 = 8$ | (f) $(-3)(1 - 5) + 2 > 10$ | (g) ${}^{100}\sqrt{-50} = -2$ | (h) $ 13 - 8 \leq 4 - 7 $ |
| (i) $-9 - (-11) - (4 - 6) = 4$ | (j) $-6(5 - 8) + 10 = -8$ | | |

Fractions are used to represent parts of whole numbers and are written in the form of $\frac{\text{numerator}}{\text{denominator}}$. These must both be whole numbers, though the denominator cannot be zero.

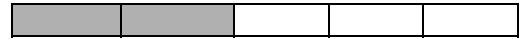
The denominator represents how many parts the whole has been divided into, and the numerator indicates the number of those parts being considered. Any whole number can be written as a fraction with a denominator of 1, so $4 = \frac{4}{1}$.

Examples

$\frac{1}{5}$ is one part of a whole divided into 5 parts.



$\frac{2}{5}$ is two parts of a whole divided into 5 parts.



$\frac{1}{2} = \frac{5}{10}$ since each part can be split further into 5 pieces. So $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$. Its simplest form is $\frac{1}{2}$.



Proper fractions are less than a whole unit and have a smaller numerator than denominator, e.g. $\frac{2}{5}$.

Improper fractions are more than a whole unit and have a larger numerator than denominator, e.g. $\frac{11}{5}$.

Mixed numbers consist of a whole part and a fraction, e.g. $3\frac{4}{5}$.

Equivalent fractions have the same simplest form, e.g. $\frac{4}{8} = \frac{3}{6}$ as both can be simplified to $\frac{1}{2}$.

We can easily convert between these types of fractions. Remember a fraction line is a division line.

$$\text{So } \frac{11}{5} = \frac{10}{5} + \frac{1}{5} = 2 + \frac{1}{5} = 2\frac{1}{5} \quad \text{and} \quad 3\frac{4}{5} = \frac{3}{1} + \frac{4}{5} = \left[\frac{3 \times 5}{1 \times 5} \right] + \frac{4}{5} = \frac{15}{5} + \frac{4}{5} = \frac{19}{5}$$

(5 parts make a whole) (3 wholes split into 5 parts each)

Practice Problems - Fractions

3. Complete the following: (a) $\frac{4}{5} = \frac{\quad}{10}$ (b) $\frac{3}{4} = \frac{\quad}{20}$ (c) $\frac{1}{3} = \frac{25}{\quad}$ (d) $\frac{3}{8} = \frac{30}{\quad}$

4. Simplify (a) $\frac{12}{16} =$ (b) $\frac{25}{40} =$ (c) $\frac{21}{39} =$ (d) $\frac{35}{49} =$ (e) $\frac{22}{33} =$

5. Convert these mixed numbers to improper fractions; and improper fractions to mixed numbers.

(a) $1\frac{2}{3} =$ (b) $2\frac{1}{2} =$ (c) $9\frac{2}{7} =$ (d) $\frac{17}{5} =$ (e) $\frac{50}{4} =$ (f) $\frac{22}{3} =$

Comparing, Adding and Subtracting Fractions

We must first rewrite each fraction as an equivalent fraction with the same denominator. Then each part of both fractions will be of the same size and the fractions can be validly compared, or added or subtracted.

To compare $\frac{4}{5}$ and $\frac{2}{3}$ we change both to fractions with a denominator of 15, then we can see

that $\frac{4}{5} = \frac{12}{15}$ is larger than $\frac{2}{3} = \frac{10}{15}$ by $\frac{2}{15}$. Adding these gives $\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15}$.

Multiplication and Division of Fractions

To **multiply** two fractions, we simply multiply their numerators and then multiply their denominators.

So $\frac{2}{5} \times \frac{1}{3} = \frac{2 \times 1}{5 \times 3} = \frac{2}{15}$ and $1\frac{3}{4} \times 2\frac{1}{5} = \frac{7}{4} \times \frac{11}{5} = \frac{77}{20} = 3\frac{17}{20}$

To **divide** fractions, change the division into a multiplication by inverting the fraction you are dividing by.

So $\frac{3}{8} \div \frac{1}{5} = \frac{3}{8} \times \frac{5}{1} = \frac{15}{8} = 1\frac{7}{8}$ and $5\frac{3}{5} \div \frac{8}{9} = \frac{28}{5} \times \frac{9}{8} = \frac{7 \times 9}{5 \times 2} = \frac{63}{10} = 6\frac{3}{10}$
 (Invert $\frac{1}{5}$ and then multiply) (Invert $\frac{8}{9}$ and multiply. See how 4 'cancels' into both 28 and 8)

Example Ted was given \$630 which was $\frac{3}{8}$ of what he was owed. How much more is he to be paid?

Let x be the total amount Ted was owed. Then $\frac{3}{8}$ of $x = \$630$, i.e. $\frac{3}{8} \times x = \$630$

So $x = 630 \div \frac{3}{8} = \frac{630}{1} \times \frac{8}{3} = \frac{210}{1} \times \frac{8}{1} = \1680 . So Ted is owed an extra $\$1680 - \$630 = \$1050$.

Practice Problems - Fractions

6. Find the value of: (a) $\frac{2}{3} + \frac{3}{4} =$ (b) $\frac{4}{7} + \frac{1}{3} =$ (c) $3\frac{1}{2} + 5\frac{3}{4} =$
(d) $\frac{3}{4} - \frac{2}{5} =$ (e) $\frac{4}{9} - \frac{3}{10} =$ (f) $4\frac{2}{5} - 2\frac{1}{10} =$ (g) $1\frac{2}{3} + \frac{1}{4} - \frac{1}{6} =$
7. Find the simplest form of: (a) $\frac{2}{3} \times \frac{5}{12} =$ (b) $\frac{7}{9} \times \frac{3}{4} =$ (c) $2\frac{1}{4} \times 3\frac{1}{5} =$
(d) $\frac{3}{4} \div \frac{1}{2} =$ (e) $\frac{5}{6} \div \frac{3}{4} =$ (f) $5\frac{2}{5} \div \frac{4}{15} =$ (g) $\frac{3}{8} \times \frac{2}{5} \div 1\frac{1}{2} =$

8. Joe earned \$1,260 in the vacation and banked $\frac{5}{7}$ of it.

- (a) What fraction was not banked? (b) How much money was banked?

9. Sarah ate $\frac{1}{3}$ of a cake one day and ate $\frac{3}{4}$ of what remained on the next day. What fraction of the cake was then left?

10. Tracy has saved $\frac{2}{5}$ of the money for her holiday. She has saved \$100. How much more does she need to save?

Decimals All real numbers can be written in decimal form, based on powers of 10. Each digit of the number has a place value corresponding to its position in the decimal, which indicates its power of 10. Zeros are used as place markers. The decimal point of a number separates the whole part of the number from the part of the number less than one.

The decimal 4583.609 is the short hand way of writing $4000 + 500 + 80 + 3 + \frac{6}{10} + \frac{0}{100} + \frac{9}{1000}$

A fraction may be converted to a decimal by dividing the numerator of the fraction by its denominator. So $\frac{3}{8}$ can be written as $3 \div 8 = 0.375$. The 3 in this decimal represents $\frac{3}{10}$. The 7 represents $\frac{7}{100}$. The 5 represents $\frac{5}{1000}$. The zero means there is no whole part in $\frac{3}{8}$.

Practice Problems - Decimals

11. Write as decimals : (a) $\frac{5}{10}$ (b) $\frac{6}{100}$ (c) $\frac{5}{100} + \frac{3}{1000}$ (d) $2000 + 5 + \frac{8}{100}$
12. Arrange from smallest to largest : 3.8 , 3 , 3.67 , 3.08 , 3.001 ; and write each as a fraction.
13. What is the value of 6 in the following? (a) 32614.8 (b) 29.0651 (c) 2 600 731 (d) 10.0046
14. Use your calculator to write as decimals: (a) $\frac{4}{5}$ (b) $\frac{5}{9}$ (c) $1\frac{1}{3}$ (d) $4\frac{3}{7}$

Some rules for decimal operations

Adding or subtracting two decimals. Be sure to line up the decimal points and corresponding digits with the same position under each other.

Multiplying by 10 or 100 or 1000, etc. Shift the decimal point to the **right** by the number of zeros. So if multiplying by 100, move the decimal point 2 places to the right.

Dividing by 10 or 100 or 1000, etc. Shift the decimal point to the **left** by the number of zeros. So if dividing by 1000, move the decimal point 3 places to the left.

Multiplying two decimals. The number of decimal places given in the original question is equal to the number of decimal places in the answer. So $0.03 \times 0.2 = 0.006$. Here two decimal places (after the decimal point) in 0.03, plus one decimal place in 0.2, makes 3 decimal places in the answer.

Dividing two decimals. First convert the denominator to a whole number.

So $0.45 \div 0.3 = \frac{0.45}{0.3} = \frac{0.45}{0.3} \times \frac{10}{10} = \frac{4.5}{3} = 1.5$ Thus $0.3 \overline{)0.45}$ is the same as $3 \overline{)4.5} = 1.5$.

Practice Problems – Decimals

15. Find the following without using a calculator. Check your answers using a calculator.

- (a) $10.48 + 8.16$ (b) $106.17 - 92.41$ (c) $11.2 - 0.8 + 3.67$ (d) 0.3×0.17
 (e) 13.06×8 (f) $(0.05)^2$ (g) 2.6721×100 (h) $3.8 \div 10$
 (i) $0.63 \div 0.07$ (j) $12.111 \div 1.1$ (k) $\frac{4.25 \times 8}{0.5}$ (l) $\frac{10(3.42 - 0.09)}{0.03}$

Percentages

Percent, %, means 'for each 100', so a percentage is a fraction of 100. For example, 23% is 23 parts of a whole split into 100 parts, i.e. $23\% = \frac{23}{100}$. So $100\% = \frac{100}{100} = 1$, the whole number.

Decimals and fractions are easy to change into percentages, by multiplying by 100%.

So $0.76 = 0.76 \times 100\% = 76\%$ and $0.083 = 0.083 \times 100\% = 8.3\%$.

Also $\frac{2}{5} = \frac{2}{5} \times \frac{100\%}{1} = \frac{200}{5}\% = 40\%$, and it is easily seen that $\frac{1}{2} = 50\%$ and $\frac{1}{10} = 10\%$.

Practice Problems - Percentages

16. Complete the following table:

Fraction	Decimal	Percentage	Fraction	Decimal	Percentage
$\frac{4}{5}$			$\frac{5}{8}$		
$\frac{1}{3}$			$\frac{3}{16}$		
	0.25			2.35	
		64%			0.3%

Examples - Using Percentages

1. Find 14% of \$325. Remember 'of' means multiply, so we calculate $\frac{14}{100} \times \frac{\$325}{1} = \$45.50$.

2. Increase 85 kg by 20%. To increase by 20%, we have the original 100% plus the new 20%, which is 120% of 85 kg in total. So the increased value will be $\frac{120}{100} \times \frac{85}{1} = 102$ kg.

3. Write 240 as a percentage of 320. This is just a fraction to be written as a percentage.

So we look at $\frac{240}{320} = \frac{3}{4}$ and then change to % $\frac{3}{4} = \frac{3}{4} \times \frac{100\%}{1} = 75\%$.

4. Find the percentage decrease when \$5400 falls to \$4725.

A percentage change is $\frac{\text{change}}{\text{original}} \times 100\%$. Here the change is a decrease of $\$5400 - \$4725 = \$675$.

So the % decrease is $\frac{\$675}{\$5400} \times 100\% = 12.5\%$.

Practice Problems – Using Percentages

17. Determine (a) 12% of 300 (b) 45% of 630 (c) 11.8 % of \$15,360
 (d) a 2% increase on 720 (e) a price of \$3500 discounted by 15% (f) 3.8% more than 54.9 km
18. Find (a) 7 kg as a percentage of 10 kg (b) 45 as a % of 54 (c) 85 cents as a % of \$4
 and the percentage change when
 (d) \$63 grows by \$21 (e) 4.5 L drops to 3.6 L (f) 3.5 km stretches to 5.6 km

Solutions to Practice Problems – Real Numbers

1. (a) 10, 3, 7, 21, 37 (b) 10, 0, 3, 7, 21, 37 (c) 10, 0, 3, 7, -2, 21, 37 (d) $-9.001, -\sqrt{7}, -1\frac{1}{4}, -\frac{3}{5}, -2$
 (e) $-9.001, 10, -1\frac{1}{4}, 0, 3, 8.94, -\frac{3}{5}, 7, -2, 21, 37$ (f) $-\sqrt{7}, \sqrt{5}$ (g) 3, 7, 37 (h) 10, 21
 2. (a) T (b) T (c) T (d) F [=8] (e) F [= -8] (f) T [$14 > 10$] (g) T (h) F [$5 \geq 3$] (i) T (j) F [=28]

Solutions to Practice Problems – Fractions

3. (a) $\frac{8}{10}$ (b) $\frac{15}{20}$ (c) $\frac{25}{75}$ (d) $\frac{30}{80}$ 4. (a) $\frac{3}{4}$ (b) $\frac{5}{8}$ (c) $\frac{7}{13}$ (d) $\frac{5}{7}$ (e) $\frac{2}{3}$
 5. (a) $\frac{5}{3}$ (b) $\frac{5}{2}$ (c) $\frac{65}{7}$ (d) $3\frac{2}{5}$ (e) $12\frac{1}{2}$ (f) $7\frac{1}{3}$
 6. (a) $\frac{17}{12} = 1\frac{5}{12}$ (b) $\frac{19}{21}$ (c) $8 + \frac{5}{4} = 9\frac{1}{4}$ (d) $\frac{7}{20}$ (e) $\frac{13}{90}$
 (f) $\frac{22}{5} - \frac{21}{10} = \frac{23}{10} = 2\frac{3}{10}$ (g) $\frac{20}{12} + \frac{3}{12} - \frac{2}{12} = \frac{21}{12} = 1\frac{9}{12}$
 7. (a) $\frac{5}{18}$ (b) $\frac{7}{12}$ (c) $\frac{36}{5} = 7\frac{1}{5}$ (d) $\frac{3}{2}$ (e) $\frac{10}{9} = 1\frac{1}{9}$ (f) $8\frac{1}{4} = 20\frac{1}{4}$ (g) $\frac{1}{10}$
 8. (a) $\frac{2}{7}$ (b) \$900 9. $\frac{2}{3}$ left after 1st day. Eating $\frac{3}{4}$ of the $\frac{2}{3}$, leaves $\frac{1}{4}$ of $\frac{2}{3} = \frac{1}{6}$ of cake left.
 10. \$150 to save to make \$250 total.

Solutions to Practice Problems – Decimals

11. (a) 0.5 (b) 0.06 (c) 0.053 (d) 2005.08 12. (a) smallest to largest 3, 3.001, 3.08, 3.67, 3.8
 12. (b) fractions $3.8 = 3\frac{8}{10}, 3 = \frac{3}{1}, 3.67 = 3 + \frac{6}{10} + \frac{7}{100} = 3\frac{67}{100}, 3.08 = 3\frac{8}{100}, 3.001 = 3\frac{1}{1000}$
 13. (a) 600 (b) $\frac{6}{10}$ (c) 600 000 (d) $\frac{6}{10000}$
 14. (a) 0.8 (b) 0.5555... (c) 3.6666... (d) 4.428571428571...
 15. (a) 18.64 (b) 13.76 (c) 14.07 (d) 0.051 (e) 104.48 (f) 0.0025
 (g) 267.21 (h) 0.38 (i) 9 (j) 11.01 (k) 68 (l) 1110

Solutions to Practice Problems – Percentages

16.

Fraction	Decimal	Percentage	Fraction	Decimal	Percentage
$\frac{4}{5}$	0.8	80%	$\frac{5}{8}$	0.625	62.5%
$\frac{1}{3}$	0.333...	$33\frac{1}{3}\%$	$\frac{3}{16}$	0.1875	18.75%
$\frac{1}{4}$	0.25	25%	$2\frac{35}{100}$	2.35	235%
$\frac{64}{100}$	0.64	64%	$\frac{3}{1000}$	0.003	0.3%

17. (a) 36 (b) 283.5 (c) \$1812.48 (d) 734.4 (e) \$2975 (f) 56.9862 km
 18. (a) 70% (b) $83\frac{1}{3}\%$ (c) 21.25% (d) $33\frac{1}{3}\%$ increase (e) 20% decrease (f) 60% increase